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1980 J. Phys. A: Math. Gen. 13 L51

(http://iopscience.iop.org/0305-4470/13/3/005)

J. Phys. A: Math. Gen. 13 (1980) L51–L53. Printed in Great Britain

LETTER TO THE EDITOR

Critical temperature for Ising systems with regularly arranged impurities†

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Received 6 November 1979

Abstract. Ferromagnetic Ising systems with four different arrangements of antiferromagnetic impurities are considered. Using the high-temperature series expansion method, T_c is determined and compared with respective values for a system containing the same amount of impurities but distributed randomly. It is shown that T_c is much more sensitive to the concentration of antiferromagnetic bonds and their strength than to the amount of frustration present in the system.

This Letter was prompted by a recent work of Longa and Oleś (1980), in which they investigated two-dimensional ferromagnetic Ising systems with antiferromagnetic impurities. In the first model every second vertical line of bonds was antiferromagnetic (the so-called Villain model), in the second (model B) every third line was antiferromagnetic and in model C every fourth was an impurity line. The authors were able to show rigorously that the magnetisation along the vertical axis can be different from zero in the B and C models, whereas Villain's model remains always in a disordered state.

The aim of this Letter is to complement those results by removing the assumption that the strength of the ferro- and antiferromagnetic interactions is the same and by adding a fourth model D, which will help us to estimate the role played by the amount of frustration present in the system. The behaviour of a system with random (quenched or annealed) distribution of ferro- and antiferromagnetic bonds has already been studied (see e.g. Matsubara and Katsura 1977, Jakubczak *et al* 1979). Since for a given concentration of antiferromagnetic bonds, p_a, there is more frustration in a system with regular than with a random distribution of impurities, it is interesting to compare both approaches. Here we wish to estimate the influence of various factors such as the amount of frustration, p_f, in the system, concentration of antiferro bonds, p_a, etc, on T_c. To this aim let us introduce four models A, B, C and D with regular arrangements of impurities. Models B and C are the same as those considered by Longa and Oleś, and model A is the completely frustrated Villain model.

The Ising spins ($S = \frac{1}{2}$) located at intersections interact with their nearest neighbours through either ferromagnetic (solid line) or antiferromagnetic (broken line) forces. The ratio of antiferro- to ferromagnetic coupling constants is denoted by x.

† Supported by a grant from the Institute for Low Temperatures and Structure Research of the Polish Academy of Sciences in Wrocław, Poland.

0305-4470/80/030051+03\$01.00

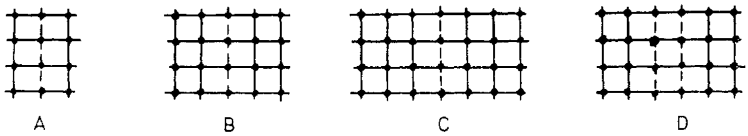

Figure 1.

On the basis of seven terms in the high-temperature series, and using linear extrapolation, we get kT_c/J_f for all four models (J_f is the ferromagnetic coupling constant). Those values, together with p_a and p_f, characterising the models A–D are given in table 1.

Table 1.

	p_a	p_f	$x=0$	$x=-0{\cdot}5$	$x=-1$	$x=-2$
A	0·25	1	1·43	0·51	0	0
B	0·143	0·66	1·87	1·63	1·28	0
C	0·1	0·50	1·97	1·81	1·64	1·30
D	0·2	0·50	1·50	0·52	0	0

We can compare them with the respective values for a random quenched system also calculated from high-temperature expansions (Pękalski and Oguchi 1975). For the annealed case T_c would be a little higher.

Table 2.

p_a \ x	0	−0.5	−1	−2
0·25	1·54	0·84	0	0
0·143	1·90	1·64	1·32	0
0·1	2·06	1·86	1·64	1·30
0·2	1·72	1·28	0·65	0

Using these two tables we can discuss how p_a, p_f and x influence T_c. As can be expected, regular arrangement of impurities lowers T_c, compared with the random system. The difference is largest for large concentration of impurities.

The case $x=0$ corresponds to a diluted ferromagnet when some vertical bonds are missing—there is of course no frustration in the system. For other (negative) values of x, there is always some frustration present. As can be seen, with growing x the value of T_c drops even more sharply than in the case of random impurities. For $x=-1$, which is a typical value taken in spin glass investigations, our results corroborate those by Longa and Oleś (1980), excluding long-range ordering in the A model. Also the D model must remain disordered. However, if we permit the same amount of impurities to be arranged at random, the system may become ordered, as is seen from table 2. Thus the way in which the impurities are arranged, specially when their concentration is not very low, is essential. It seems however that the freedom to move and form clusters of like bonds is more important in pushing the T_c up than the reduction of the number of frustrated plaquettes. This can be seen from comparison of the models A, C and D. The models C and D have the same value of p_f, which is half that of A; still, the T_c values are close for the fully frustrated model A and 'half-frustrated' model D, and different for the model C, the reason being similar p_a values for the A and D models but a different one for the C model.

Apart from the strength of antiferromagnetic interactions, the other factor influencing T_c is the concentration p_a of impurity bonds. This is best seen upon inspecting T_c for the C and D models, which have the same p_f but different p_a. If, for $x=-1$, we allow the impurities to be arranged at random, or to comply with thermodynamic equilibrium requirements, both models have non-zero transition temperature. Fixing impurities into the respective C or D patterns eliminates the possibility of long ordering in the D model only, since it contains twice as many antiferro- bonds as the model C. The difference between the random and regular systems with impurities grows, for a fixed x, with p_a and, for a given p_a, it increases with x.

It seems therefore that the decisive role in determining whether a ferromagnetic Ising system containing antiferromagnetic bonds as impurities can be magnetically ordered is played by:

(1) the concentration of impurities;

(2) the strength of the impurity couplings;

(3) the way the impurities are distributed (annealed, quenched or in a regular pattern).

The amount of frustration present in the system is of only secondary importance, at least as far as the determination of T_c is concerned.

I am grateful to Dr A M Oleś for a valuable discussion.

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